

Character table for D_4

$$D_4 = \langle r, j \rangle, \quad o(r) = 4, \quad o(j) = 2, \quad rj = jr^{-1}$$

8	1	2	1	2	2
	e	r	r ²	j	jr
χ_1	1	1	1	1	1
χ_2	1	1	1	-1	-1
χ_3	1	-1	1	1	-1
χ_4	1	-1	1	-1	1
χ_5	2	0	-2	0	0

$$[D_4, D_4] = \{e, r^2\} \Rightarrow D_4/[D_4, D_4] = \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$|G| = \sum_{k=1}^s d_k^2, \quad d_k = \dim \chi_k = \chi_k(e)$$

$$8 = 1^2 + 1^2 + 1^2 + 1^2 + 2^2$$

$$\langle \underbrace{\chi_1 + \chi_2 + \chi_3 + \chi_4}_{2 \quad 0 \quad 2 \quad 0 \quad 0}, \chi_5 \rangle = 0 \quad \langle \chi_5, \chi_5 \rangle = 1$$

χ_5 is character of

$$\varphi: D_4 \longrightarrow GL_2(\mathbb{R}) \leq GL_2(\mathbb{C})$$

$$\varphi_r = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \varphi_s = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$